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FLOW INSTABILITIES IN PARALLEL-CHANNEL FLOW SYSTEMS OF GAS–LIQUID TWO-PHASE MIXTURES

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Abstract—Parallel-channel two-phase flow systems with compressible capacities and the negative-slope characteristics of the pressure drop vs flow rate curve suffer from a non-uniform flow distribution and/or pressure drop oscillation, depending on the pressure drop characteristics of each channel, under certain operating conditions. The modes of flow distribution are quite similar to those observed in boiling channels. Pressure drop oscillations are subdivided into two types: relaxation oscillation and quasi-static oscillation. The modes of oscillation observed in the parallel-channel system are a single-channel mode, a U-tube mode and a multi-channel mode. These modes of oscillation are closely related to the type of oscillation. Upstream compressibility in the gas feed pipe has a destabilizing effect on the oscillation. On the other hand, upstream compressibility in the liquid feed pipe has a stabilizing effect on the oscillations of the gas flow rate and the pressure drop between the headers of parallel channels, but induces a small oscillation of the liquid flow rate. These non-uniform flow distribution and oscillation patterns are analyzed; the results of the analysis are found to be in good agreement with the experimental results. Thus, the characteristics of the non-uniform flow distribution and the oscillation can be estimated by the method of analysis presented in this paper.

Key Words: flow instability, two-phase flow, parallel channels, pressure drop oscillation, flow distribution

1. INTRODUCTION

In the past two decades, flow instability problems have been widely investigated in boiling two-phase flows in the development of BWR, PWR and FBR systems (Bouré *et al.* 1971; Lahey & Drew 1980). Most research into flow instability has been focused upon the "density wave oscillation". Density wave oscillations are not limited to systems of nuclear reactors. They appear in all types of boilers, chemical plants, air conditioning units and many other gas and liquid pipelines. In these systems, there are not only density wave oscillations but also other types of flow instability, such as pressure drop oscillations, geysering and chugging. Under certain conditions, these latter types of instability may lead to more severe operational and safety problems than would be caused by the density wave oscillation.

Among these flow instabilities, pressure drop oscillations are characterized by the existence of a negative slope in plots of pressure drops vs flow rates and by a compressible capacity; they represent one of the typical non-linear oscillations expressed in van der Pol's equation (Ozawa *et al.* 1982). Although some authors described the mechanism and also the threshold conditions of pressure drop oscillations in a single-channel system (Stenning & Veziroglu 1965; Maulbetsch & Griffith 1966), the characteristics of pressure drop oscillations in a parallel-channel system have hardly been reported so far.

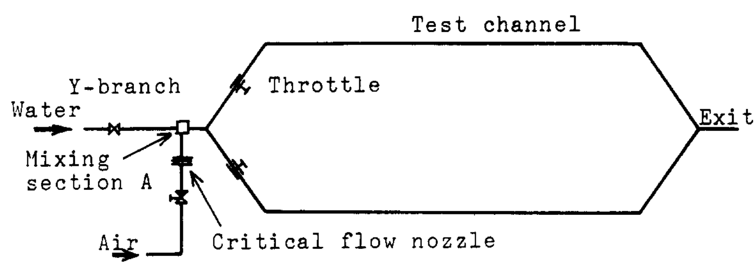
In a parallel-channel system with the negative-slope characteristics of the pressure drop vs flow rate curve, not only a pressure drop oscillation but also a non-uniform flow distribution takes place under certain operating conditions, the latter phenomenon also causing severe safety problems, such as dryout of the channel. This report describes the experimental and analytical investigations of the flow distribution and the flow oscillation in a parallel-channel two-phase flow system, and the fundamental characteristics and a simple method to estimate these flow instabilities are discussed.

In this study, small diameter tubes were used for test channels. This was mainly in order to realize the negative-slope characteristics of the pressure drop vs flow rate curve and pressure drop oscillations. As is well-known, there are many equipments and pipeline systems with mixing of two phases and flow distribution of two-phase mixtures. In such systems, compressible capacities will also exist. When these systems have the negative-slope characteristics of the pressure drop vs flow rate curve, they will suffer from a drastic flow distribution and a pressure drop oscillation. Thus, the present results will contribute to the fundamental understanding of two-phase flow dynamics. Moreover, the present results will be directly applied to pipelines, such as air conditioning units will smaller diameter tubes.

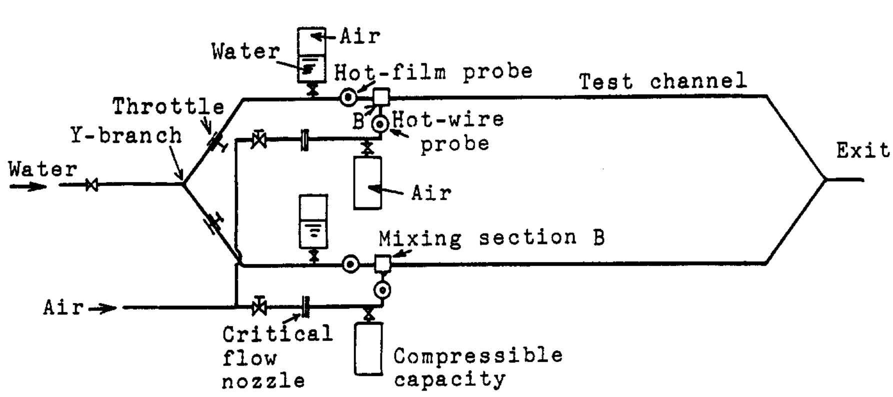
2. EXPERIMENTAL APPARATUS

The experimental apparatus is shown schematically in figures 1(a, b). The main parts of this system are water feed pipes, air feed pipes, mixing sections, parallel channels of 3.1 m length and 3.1 mm i.d. glass tubes, a Y-branch section and compressible capacities in the air and water supply feed pipes. Air from a compressor and water from an overflowing tank located some 15 m above the system were supplied to the mixing sections. The compressible capacities in the air feed pipes were provided by air tanks, and those in the water feed pipes were also provided by tanks containing water and air above the free level of water. Therefore, in the compressible capacities of the air feed pipes, the air was accumulated or ejected depending on the pressure difference between the tank and the air feed pipes. In the compressible capacities of the water feed pipes, the water was accumulated or ejected depending on the difference between the pressure in the water feed pipe and the pressure of the air enclosed in the tank.

Experiments were conducted in two series: the flow distribution experiment and the flow oscillation experiment. In the flow distribution experiment [see figure 1(a)] a two-phase mixture was formed in the mixing section designated by "A" and flowed through the Y-branch section. In this case, the compressible capacities were isolated from the channels. In the flow oscillation experiments [see figure 1(b)], a two-phase mixture was formed separately in each mixing section B in both channels after the water flow was distributed at the Y-branch. Then the air was supplied not to the mixing section A but to each mixing section B. In this case, the compressible capacities were connected to the channels.



(a)



(b)

Figure 1. Experimental apparatus.

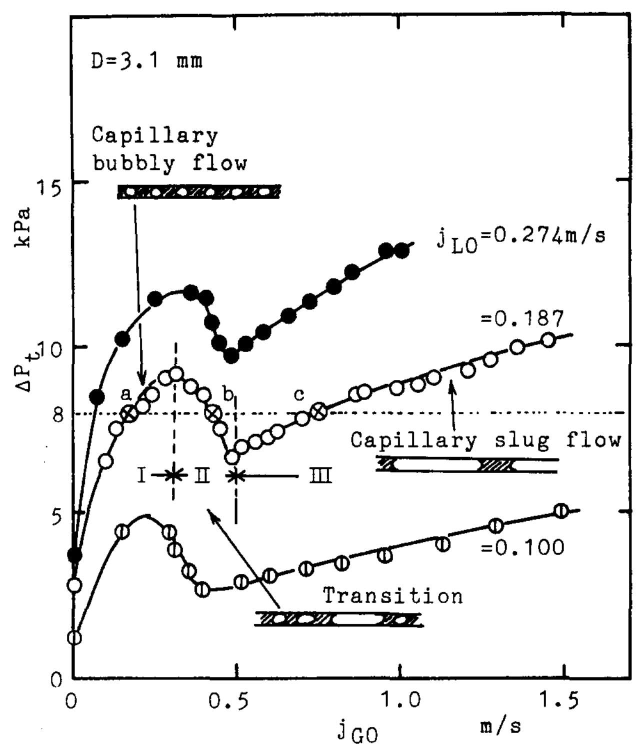


Figure 2. Pressure drop characteristics.

The inner walls of the parallel channels were sufficiently smooth so that the friction factor agreed well with that of the Hagen–Poiseuille equation for a laminar flow and the Blasius equation for a turbulent flow. The water flow rate to the system was kept constant against any change of flow downstream. The air flow rates were also kept constant upstream of the compressible capacities.

3. PRESSURE DROP CHARACTERISTICS AND FLOW OSCILLATIONS IN A SINGLE CHANNEL

In figure 2, typical examples of the pressure drop characteristics in a single channel are shown against the volumetric gas flux, j_{G0}. They clearly show the negative-slope characteristics of the pressure drop vs the volumetric gas flux. The plot of the pressure drop vs the volumetric gas flux is divided into three regions. In the first region I, relatively short bubbles of the same order of length as the tube diameter flow at approximately uniform intervals and at uniform velocity. This flow pattern is called "capillary bubbly flow". In the region of relatively high volumetric gas flux III, the bubble length becomes very large compared with the tube diameter, almost reaching 1 m in a certain case. This flow pattern is called "capillary slug flow" or "annular flow". The middle region II, which is the region with negative-slope characteristics, is a region of transition between flow patterns from regions I to III. The pressure drop characteristics are closely related to the flow pattern in the capilliary tube (Suo & Griffith 1963; Ozawa *et al.* 1979b).

The pressure drop characteristics differed slightly in the two channels making up the parallel-channel system. The difference was small enough to be neglected in the flow distribution problem, but caused some divergences in oscillation periods and amplitudes between the two channels in the flow oscillation experiments.

These pressure drop–volumetric gas flux characteristics with a negative slope are analogous to the pressure drop vs total mass flux characteristics observed in a boiling channel of a relatively small diameter tube (Akagawa *et al.* 1971; Ozawa *et al.* 1979a, c). It is well-known that these adiabatic and/or diabatic flow systems with negative-slope characteristics and upstream compressible capacity show pressure drop oscillations (Stenning & Veziroglu 1965; Maulbetsch & Griffith 1966; Ozawa *et al.* 1979a–c). The region characterized by a negative slope is unstable and exhibits flow excursion. Thus, the state is far from the normal flow condition. However, this flow excursion is limited to a certain extent by the accumulation capacities of mass and momentum and the positive slope of the plot of the pressure drop vs the volumetric gas flux in regions I and III. Compressibility returns the state of flow excursion to normal flow conditions. These two processes alternate, the oscillation being sustained. This fundamental mechanism of pressure drop oscillation is analogous to a non-linear oscillator, as investigated by van der Pol (1926), and the fundamental equation of this pressure drop oscillation can be approximated by van der Pol's equation (Ozawa *et al.* 1971a, b, 1982).

In the adiabatic gas–liquid flow system with upstream compressibilities in gas and liquid feed pipes there are two types of pressure drop oscillation: relaxation oscillation and quasi-static oscillation.

Relaxation oscillations are characterized by the relaxation process (flow excursion) between two asymptotically steady states as shown schematically by curves (a) and (b) in figure 3; i.e. the state point in a space represented by the axes of the pressure drop, ΔP_t, the volumetric gas flux, j_{G0}, and the volumetric liquid flux, j_{L0}, during oscillation moves approximately on the static characteristic surface in regions I and III, away from the surface in region II. This flow excursion can be considered a catastrophe on a cusp surface. In case (a), the locus of the state point on the j_{G0}–j_{L0} plane is almost parallel with the j_{G0} axis, i.e. the volumetric liquid flux is kept constant during the oscillation. In case (b), a small change in j_{L0} occurs during oscillation, the change in the volumetric liquid flux being almost in phase with the change in the volumetric gas flux. This type of oscillation, (b), occurs in systems with relatively high compressibility in the gas feed pipe and relatively little compressibility in the liquid feed pipe (Ozawa *et al.* 1982).

Quasi-static oscillation is shown schematically in curve (c) in figure 3. The amplitude of the liquid flow during the oscillation is larger than that of the relaxation oscillation, because of the relatively high compressibility in the liquid feed pipe. During the quasi-static oscillation, the state point moves approximately on the static characteristic surface. The locus of the state point on the j_{G0}–j_{L0} plane is elliptic owing to the phase shift between the change in the volumetric gas flux and the volumetric liquid flux. This type of oscillation occurs in a system with relatively high compressibility in the liquid feed pipe (Ozawa *et al.* 1982).

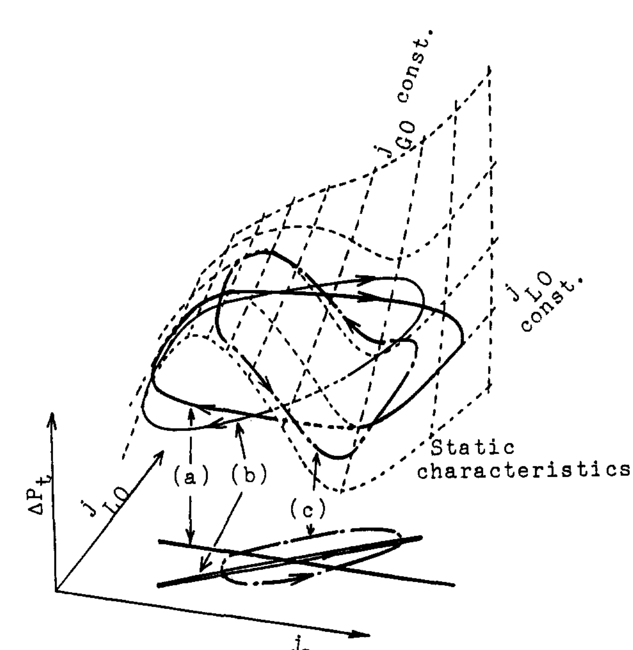


Figure 3. Locus of the oscillation.

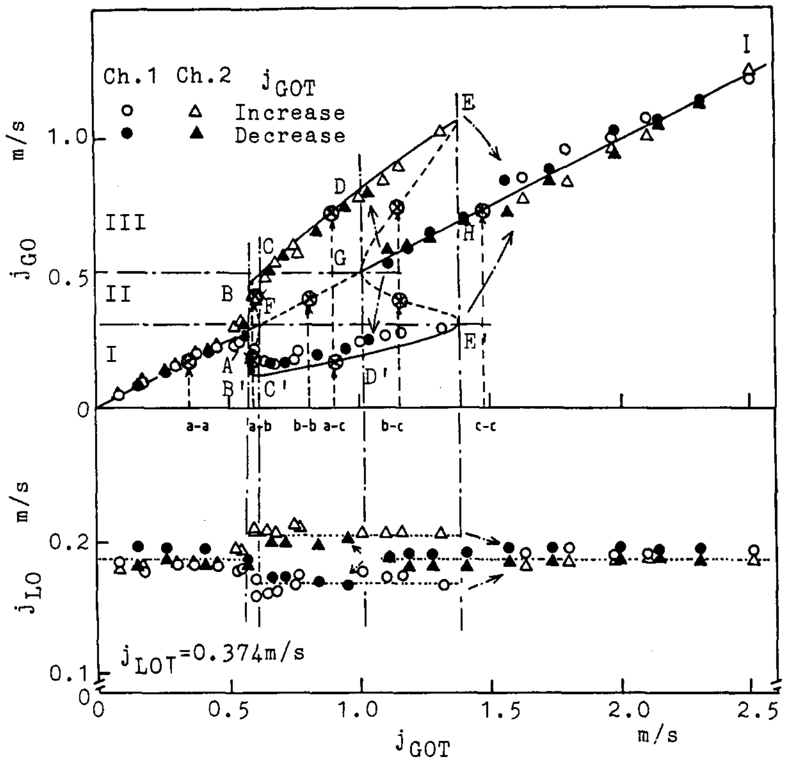


Figure 4. Flow distribution characteristics.

4. FLOW DISTRIBUTION

4.1. Flow distribution characteristics

This section deals with the flow distribution characteristics in the parallel-channel system as determined from the results obtained in the first series of experiments.

The flow distribution diagrams for the total volumetric liquid flux, $j_{L0T} = 0.374$ m/s, are shown in figure 4. The mean volumetric liquid flux in each channel is $j_{L0} = 0.187$ m/s, and the pressure drop–volumetric gas flux characteristics of each channel are shown in the curve of $j_{L0} = 0.187$ m/s in figure 2.

Various modes of flow distribution were observed in the parallel-channel system as a function of the ΔP_t–j_{G0}–j_{L0} characteristics in each channel. When the total volumetric gas flux, j_{G0T}, was increased successively, the gas flow rates in the first region ($j_{G0T} < 0.5$) were uniformly distributed in both channels, but at about $j_{G0T} = 0.5$ m/s the gas flow rates began to be non-uniformly distributed, the non-uniformity of the flow rates between the two channels growing with the increase in j_{G0T}. When the volumetric gas flux exceeded 1.5 m/s, the gas flow was uniformly distributed again.

When the volumetric gas flux, j_{G0T}, was decreased successively from a higher value, e.g. point I, the uniform flow distribution continued up to about $j_{G0T} = 1.1$ m/s. In this region, there was hysteresis in the flow distribution. Below this value, the flow distribution became non-uniform up to about $j_{G0T} = 0.5$ m/s. The flow distribution in this region coincided with that found during the rise in j_{G0T}. Below this total volumetric gas flux value ($j_{G0T} = 0.5$ m/s), the distribution returned to a uniform pattern. These distributions characteristics were quite similar to those observed in a parallel-channel boiling system with the negative-slope characteristics of the pressure drop vs flow rate curve (Akagawa *et al.* 1971; Ozawa *et al.* 1979c).

The change in volumetric liquid flux in each channel, j_{L0}, plotted against an increase in and a decrease in total volumetric gas flux is also shown in figure 4. Comparison of the flow distribution diagrams of the gas flow and the liquid flow shows that, with a non-uniform distribution of the gas flow, the liquid flow is also uniformly distributed and that, with a non-uniform distribution of the gas flow, the liquid flow is also non-uniformly distributed. The volumetric liquid flux in the channel with the higher volumetric gas flux of the two is also higher than that in the other channel, which may be due to the shear force of the gas phase acting on the liquid phase.

It is difficult to determine all the characteristics of the flow distribution of gases and liquids without having information about the phase interaction in the Y-branch section. Therefore, we will now discuss only the gas flow distribution. Neglecting the small variation in the liquid flow during the experiments (see figure 4), the gas flow distribution can be constructed easily from the static characteristics shown in figure 2 in the following manner (Akagawa *et al.* 1971). For a given value of ΔP_t, for instance $\Delta P_t = 8$ kPa, three values of j_{G0} (points a, b and c) are determined on the ΔP_t–j_{G0} curve for $j_{L0} = 0.187$ m/s in figure 2. This means that six combinations of the volumetric gas flux are possible in parallel-channel systems, i.e. a–a, a–b, a–c, b–b, b–c and c–c. Thus, six points marked $\otimes$ can be plotted on the j_{G0}–j_{G0T} plane for a given ΔP_t value. Changing the ΔP_t value successively produces the solid and dashed lines indicating the flow distribution characteristics.

The j_{G0} axis is divided into three parts corresponding to regions I, II and III in figure 2. The flow distribution curve 0–F then corresponds to the combination of states I and I in each channel, respectively. The curves F–C and F–C′ correspond to II and I, and so on. These relationships between the characteristic curves of the flow distribution in figure 4 and the pressure drop characteristics of each channel in figure 2 are listed in table 1.

From a comparison of the experimental results and the characteristic curves it can be seen that almost all data points fall approximately on the solid lines and that there are no data coinciding with the dashed lines. This means that not all flow distribution characteristics obtained from the static characteristics of ΔP_t–j_{G0} can be realized in the experiment. To estimate the actual flow distribution we should have some kind of criterion for the stability of the flow distribution.

4.2. *Analysis of the flow distribution characteristics*

The flow distribution characteristics in parallel-channel systems can be obtained from the static characteristics of ΔP_t–j_{G0} as described in the previous section. However, this does not show whether the flow condition is stable or not. If the stability of the flow distribution is to be known, the dynamic behavior of parallel-channel flow must be analyzed, for the flow distribution and its stability are closely related to the flow excursion phenomena in parallel channels. In this section, a simplified stability analysis of the gas flow distribution is presented under the assumption of a uniform distribution of the liquid flow.

The flow model is composed of gas and liquid feed pipes, the mixing section "A", the branch section and parallel channels, as shown in figure 1(a). The total volumetric gas flux in the system, j_{G0T}, and the pressure at the channel exit, P_e, are kept constant during the transients. The volumetric liquid flux in all channels is assumed to be constant against any change in the volumetric gas flux. The residence time of the flow in the channel is relatively short compared to the time constant of the transients observed in the experiment. The dynamic behavior of the system thus can be expressed approximately as a lumped-parameter system. In this lumped-parameter system, the volumetric gas and liquid fluxes are used as representative velocities.

The mass balance of the gas phase is written as

$$\rho_G F_t j_{G0T} \equiv \rho_G F_t j_{G1} + \rho_G F_t j_{G2} = \text{const.} \tag{1}$$

Table 1. Relationship between flow distribution characteristics and pressure drop characteristics

Flow distribution characteristics	Pressure drop characteristics in the single channel
0–F	I–I
F–C, F–C′	II–I
C–E, C′–E′	III–I
F–G	II–II
G–E, G–E′	III–II
G–I	III–III

The velocity of the two-phase mixture is given by $j_G + j_{L0}$, and the momentum balance in each channel is written as

$$P_i - P_e = \frac{d}{dt}\{\rho_{M1} L(j_{G1} + j_{L0})\} + \Delta P_{t1} \qquad [2]$$

and

$$P_i - P_e = \frac{d}{dt}\{\rho_{M2} L(j_{G2} + j_{L0})\} + \Delta P_{t2}, \qquad [3]$$

where ρ_{M1} and ρ_{M2} are the mean densities of two-phase mixtures, and are assumed to be constant in the neighborhood of given steady-state values of j_{G0} and j_{L0}. ΔP_{t1} and ΔP_{t2} are the frictional pressure drops in each channel.

If the flow pattern in the test channel was a separated flow, the inertia term could be written as $d/dt\{\rho_G j_G + \rho_L j_{L0}\}$. Then, under the assumption of constant j_{L0}, only the inertia of gas, which is so small as to be negligible, would be taken into account in the analysis. In the present system, however, the flow pattern was not separated flow, but as indicated in figure 2. Therefore, both the liquid and the gas in the test channel are accelerated together by any change in j_G. Thus, the inertia term is expressed as a function of the mixture density, ρ_M, and the mixture velocity, $j_G + j_{L0}$.

Eliminating j_{G2} and j_{L0} from [1]–[3], we obtain the following:

$$\frac{d}{dt}(Mj_{G1}) = \Delta P_{t2} - \Delta P_{t1}, \qquad [4]$$

where the inertial mass is defined as $M \equiv (\rho_{M1} + \rho_{M2})L$.

Here we describe the qualitative discussion of the characteristics of [4]. In the case where both channels are in a state in which the curve of the pressure drop vs the volumetric gas flux has a positive slope, when ΔP_{t2} happens to become larger than ΔP_{t1} as a result of some flow perturbation, the r.h.s. of [4] becomes positive and j_{G1} increases with time, while j_{G2} decreases because the total volumetric gas flux is kept constant. Then, ΔP_{t1} increases and ΔP_{t2} decreases, which leads to the decrease in the difference between ΔP_{t1} and ΔP_{t2}. The state thus approaches stable conditions.

In the case where both channels are in the state with negative-slope characteristics, when ΔP_{t2} happens to become larger than ΔP_{t1}, j_{G1} increases with time and j_{G2} decreases. Then, ΔP_{t1} decreases and ΔP_{t2} increases according to the negative-slope characteristics, which further enhances the difference in pressure drops. Thus, j_{G1} increases more and more and the flow excursion takes place.

Finally, we discuss the case of channel 2 being in the state with positive-slope characteristics and channel 1 being in the state with negative-slope characteristics. As ΔP_{t2} exceeds ΔP_{t1}, j_{G1} increases with time and j_{G2} decreases. Then both ΔP_{t1} and ΔP_{t2} decrease. If the decrement of ΔP_{t2} against the variation of j_{G2} is larger than that of ΔP_{t1} against the variation of j_{G1}, the difference between ΔP_{t1} and ΔP_{t2} becomes smaller and approaches zero under a certain condition. On the other hand, if the decrement of ΔP_{t2} is smaller than that of ΔP_{t1}, the difference becomes larger, which leads to a flow excursion.

Now we derive the stability criterion of the flow distribution. We define the following Liapunov function, W:

$$W \equiv \frac{M\left(\frac{dj_{G1}}{dt}\right)^2}{2}. \qquad [5]$$

Differentiating [5] with respect to time, t, produces

$$\frac{d^2 j_{G1}}{dt^2} = -\left\{\frac{d(\Delta P_{t1})}{dj_{G1}} + \frac{d(\Delta P_{t2})}{dj_{G2}}\right\}\left(\frac{dj_{G1}}{dt}\right) M. \qquad [6]$$

Substituting [6] by the time derivative of function, W, leads to

$$\frac{dW}{dt} = -\left\{\frac{d(\Delta P_{t1})}{dj_{G1}} + \frac{d(\Delta P_{t2})}{dj_{G2}}\right\}\left(\frac{dj_{G1}}{dt}\right)^2. \qquad [7]$$

The function, dW/dt, becomes negative when the following inequality is satisfied:

$$\frac{d(\Delta P_{t1})}{dj_{G1}} + \frac{d(\Delta P_{t2})}{dj_{G2}} > 0. \qquad [8]$$

According to Liapunov's direct method (Denn 1975), [8] furnishes the condition for stability, which coincides with that proposed by Akagawa *et al.* (1971) based on a linearized stability theory.

The flow distribution characteristics represented by the solid lines are estimated to be stable conditions by means of [8]. On the other hand, the dashed lines are estimated to be unstable conditions. For instance, the values of $d(\Delta P_t)/dj_G$ for the points a, b and c in figure 2 are 14.4, -16.2 and 4.7, respectively. Then, the l.h.s. of [8] has negative values for the combinations of a–b, b–b and b–c. Thus, the points designated by the mark ⊗ for a–b, b–b and b–c in figure 4 are estimated to be unstable conditions. As the volumetric liquid differs from the mean value, there is a discrepancy between the experimental results and the criterion of [8], especially on the dashed lines B–F and B′–F. However, most of the experimental results confirm the validity of this stability criterion of the flow distribution. The applicability of [8] has also been confirmed in parallel-channel boiling systems (Akagawa *et al.* 1971; Ozawa *et al.* 1979c).

5. FLOW OSCILLATION IN A PARALLEL CHANNEL

In this section, flow oscillation problems are discussed on the basis of the experimental results of the second series and the analytical results of the parallel-channel two-phase flow system.

5.1. *Type and mode of oscillation*

Typical experimental results of flow oscillation and the oscillation of the pressure drop in the parallel-channel system are shown in figures 5(a–d). These traces were obtained under identical operating conditions of j_{G0}, j_{L0}, V_G and V_L in the two channels, i.e. $j_{G01} = j_{G02}$, $j_{L01} = j_{L02}$, $V_{G1} = V_{G2}$ and $V_{L1} = V_{L2}$. The pressure drop oscillations in the parallel-channel system were also classified into two types, as observed in the single-channel system: relaxation oscillation and quasi-static oscillation. The oscillations in figure 5(a) are relaxation oscillations and those in figures 5(c, d) are quasi-static oscillations. The oscillation in figure 5(b) is a combination of relaxation and quasi-static oscillations. The fundamental features of these oscillations are similar to those in the single-channel system. A relaxation oscillation occurs when there are relatively large compressible capacities in the gas feed pipes; for instance, V_{G1} and V_{G2} are 3×10^{-3} m^3 and V_{L1} and V_{L2} are zero, as shown in figure 5(a). On the other hand, a quasi-static oscillation occurs when there are relatively large compressible capacities in the liquid feed pipes, for instance, V_{L1} and V_{L2} are 2–4 $\times 10^{-3}$ m^3, as is shown in figures 5(c, d).

The types and modes of oscillation are closely interrelated, as are summarized in table 2. The oscillation modes in the parallel-channel system are classified in the following three groups with reference to the phase relation between the two channels:

(1) *Single-channel mode [figure 5(a)].* The type of oscillation in each channel is a relaxation oscillation. The gas flows in each channel oscillate in phase with each other. The period and the amplitudes of the gas flow and the pressure drop are relatively large. The amplitude of the liquid flow oscillation is very small. This mode of oscillation is observed for relatively large values of V_{G1} and V_{G2} and very small values of V_{L1} and V_{L2}.

Table 2. Oscillation modes and types

Mode	ΔP_T:	Oscillation	Constant	
Type	Flow:	In phase	Different frequencies	180° Out of phase
Relaxation		Single-channel	—	—
Quasi-static		—	Multi-channel	U-tube

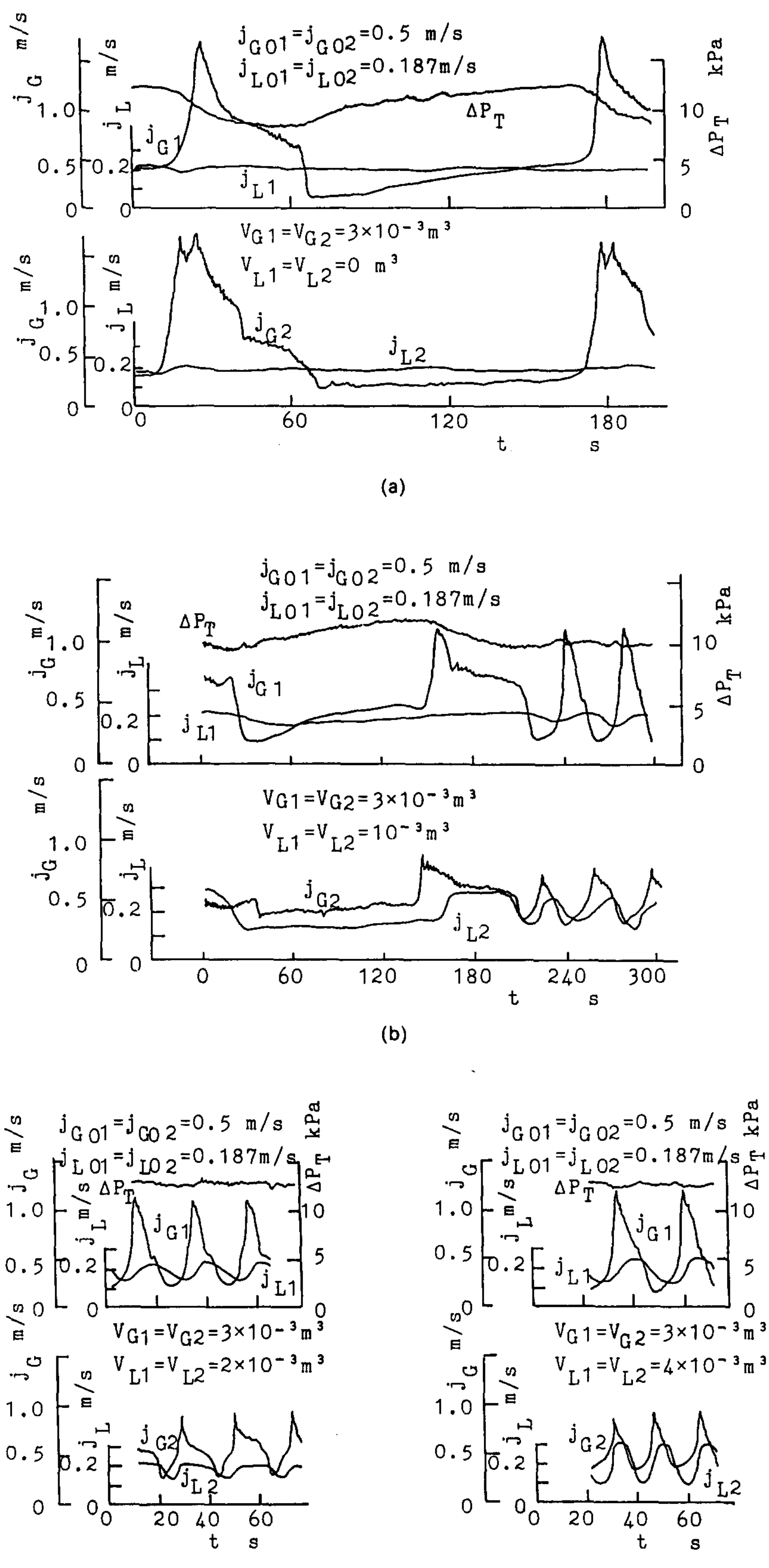


Figure 5. Oscillation traces: (a) single-channel mode (relaxation oscillation); (b) transition; (c) U-tube mode (quasi-static oscillation); (d) multi-channel mode (quasi-static oscillation).

(2) *U-tube mode [figure 5(c)].* The oscillation is a quasi-static oscillation. The gas flow and the liquid flow oscillate 180° out of phase in the two channels. The period and the amplitude of the gas flow oscillation are relatively small compared with those of the single-channel mode. However, the amplitude of the liquid flow oscillation is relatively large compared with that of the single-channel mode. The pressure drop between the headers, ΔP_T, is approximately constant during the oscillation. This oscillation occurs in the case of relatively large values of V_{L1} and V_{L2}.

(3) *Multi-channel mode [figure 5(d)].* The oscillation is a quasi-static oscillation. The gas flow in each channel oscillates independently, i.e. the oscillation period in one channel differs from that in the other. The oscillation profiles have features similar to those in the U-tube mode. The magnitudes of V_{L1} and V_{L2} in this mode are larger than those in the U-tube mode when the compressible capacities of the two channels are identical. This mode of oscillation is caused by the difference in flow rates, compressible capacities and the pressure drop characteristics between the two channels.

Figure 6 is the oscillation mode map on the V_G–V_L plane for the case of equally large compressible capacities in both channels. It is clearly seen that the mode of oscillation in a parallel-channel system is determined by the magnitudes of the compressible capacities in the gas and liquid feed pipes. There is a transition region between the single-channel mode and the U-tube mode. In this region, both modes of oscillation are observed, as seen in figure 5(b).

5.2. Period and amplitude of oscillation

In this section, the characteristics of the oscillations observed in this experiment are discussed with reference to the period and the amplitude of oscillation of the gas and liquid flows and the pressure drop between the headers. The experimental data presented are those obtained under the conditions of $V_{G1} = V_{G2}$ and $V_{L1} = V_{L2}$.

The period of oscillation, T, is shown against the compressible capacity of the gas feed pipes, V_G, in figure 7. Data for $V_L = 0$ are those of the relaxation oscillation discussed in the previous

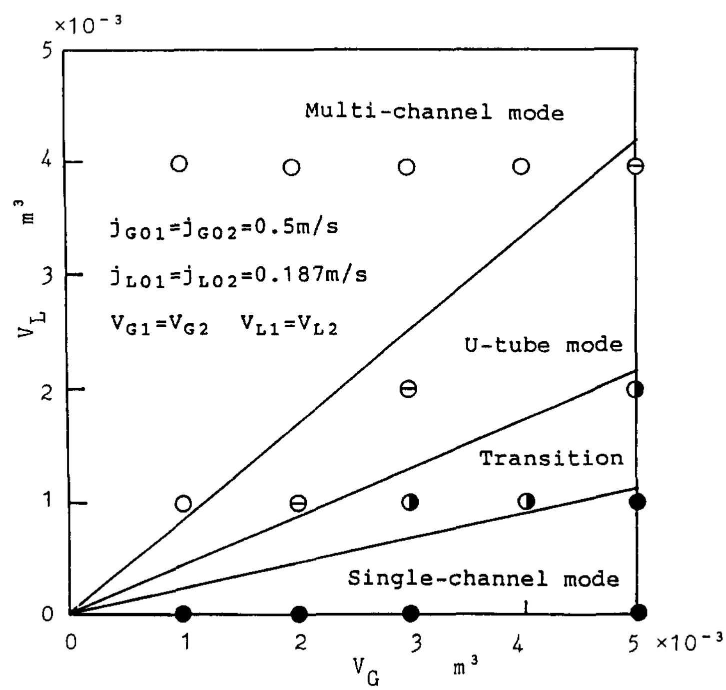


Figure 6. Oscillation mode map.

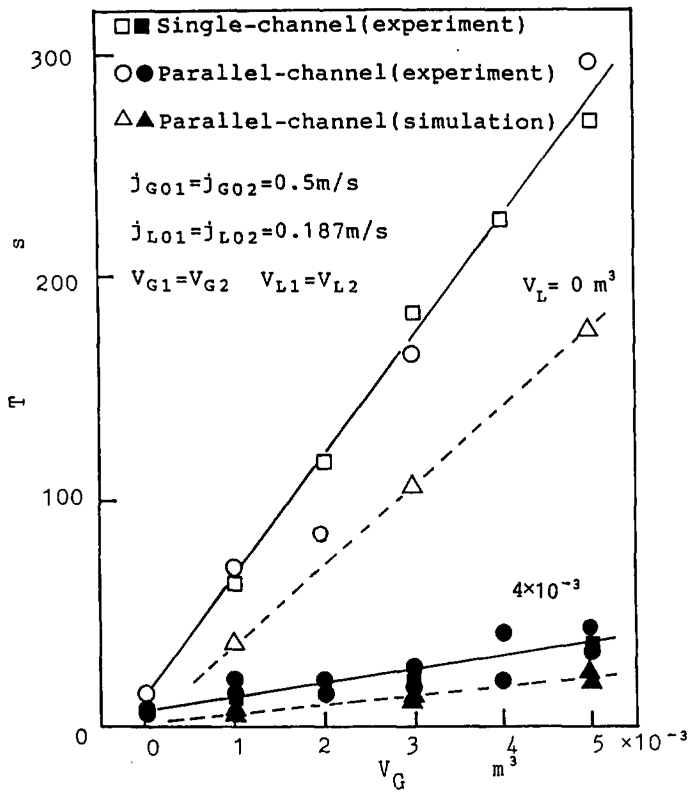


Figure 7. Period of oscillation.

section, and the data for $V_l = 4 \times 10^{-3}$ m^3 are those of the quasi-static oscillation. The period of the relaxation oscillation is, in general, longer than that of a quasi-static oscillation. In both cases, the period of oscillation increases approximately linearly with an increase in the compressible capacities, V_G. Data obtained in the single-channel experiment are almost identical to those for the parallel-channel system.

In our previous work (Ozawa *et al.* 1982) on oscillatory flow instability in the single-channel system, the behavior of the relaxation oscillation was expressed approximately by van der Pol's equation:

$$\frac{d^2X}{d\tau^2} - \epsilon(1 - X^2)\frac{dX}{d\tau} + X = 0, \tag{9}$$

where X and τ represent a dimensionless volumetric gas flux and a dimensionless time, respectively, and ϵ is a dimensionless parameter which characterizes a property of the limit cycle oscillation. The period of the oscillation expressed by [9] is, in general, affected significantly by the parameter, ϵ, and is approximated by a linear function of ϵ. The dimensionless time, τ, and the parameter, ϵ, were expressed (Ozawa *et al.* 1982) as

$$\tau = \frac{t}{\sqrt{\dfrac{M\left(1 + \dfrac{C_G}{C_L}\right)}{C_G}}} \tag{10}$$

and

$$\epsilon = -\frac{\left\{\dfrac{d(\Delta P_t)}{dj_{G0}} + \dfrac{C_G}{C_L}\dfrac{d(\Delta P_t)}{dj_{L0}}\right\}}{\sqrt{M\left(1 + \dfrac{C_G}{C_L}\right)C_G}}, \tag{11}$$

where

$$C_{\mathrm{L}} = \frac{P_{\mathrm{L}}F_{\mathrm{LC}}}{V_{\mathrm{L}}} \quad \text{and} \quad C_{\mathrm{G}} = \frac{P_{\mathrm{G}}F_{\mathrm{GC}}}{V_{\mathrm{G}}}.$$

Thus the period of oscillation, T, is expressed as

$$T \propto -\frac{\left\{\dfrac{\mathrm{d}(\Delta P_{\mathrm{t}})}{\mathrm{d}j_{\mathrm{G0}}}\right\}}{C_{\mathrm{G}}} - \frac{\left\{\dfrac{\mathrm{d}(\Delta P_{\mathrm{t}})}{\mathrm{d}j_{\mathrm{L0}}}\right\}}{C_{\mathrm{L}}}. \qquad [12]$$

Now we consider the case of $V_{\mathrm{L}} = 0$. The period is now expressed only by the first term of [12]. The term $-\{\mathrm{d}(\Delta P_{\mathrm{t}})/\mathrm{d}j_{\mathrm{G0}}\}$ represents the gradient of change in the pressure drop with respect to the change in the volumetric gas flux. The quantity, C_{G}, represents the rate of pressure change for the change in the volumetric gas flux, as is seen in [22]. Therefore, the quantity, $-\{\mathrm{d}(\Delta P_{\mathrm{t}})/\mathrm{d}j_{\mathrm{G0}}\}/C_{\mathrm{G}}$, represents the time elapsed until the necessary change in state (change in the volumetric gas flux). When V_{G} is small, the rate of pressure change for the change in gas flow into the capacity becomes high. On the other hand, the rate of pressure change for the change in gas flow becomes low for a large capacity. Then the rate of state change is high in a system with a small capacity of the gas feed pipe. In a system with a large capacity, the rate of state change becomes low. Thus the period of oscillation is affected significantly by the magnitude of the compressible capacity and is a linear function of V_{G}.

In the system with compressible capacities in both the gas and liquid feed pipes, the second term of [12] has a significant effect on the period. In the region where the pressure drop oscillation occurs, the value $\{\mathrm{d}(\Delta P_{\mathrm{t}})/\mathrm{d}j_{\mathrm{G0}}\}$ is negative, but the value $\{\mathrm{d}(\Delta P_{\mathrm{t}})/\mathrm{d}j_{\mathrm{L0}}\}$ is always positive, as is seen in figure 2. This means that the effect of the compressible capacity in the liquid feed pipe on the period of oscillation is opposite to that in the gas feed pipe. Consequently, the period of oscillation increases with V_{G} under a constant value of V_{L}, and decreases with V_{L} under a constant value of V_{G}.

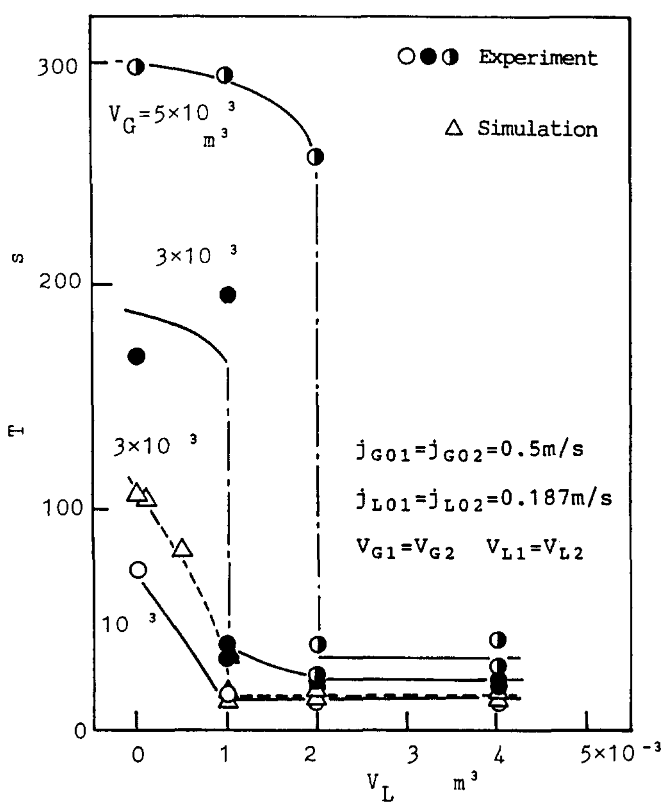


Figure 8. Period of oscillation.

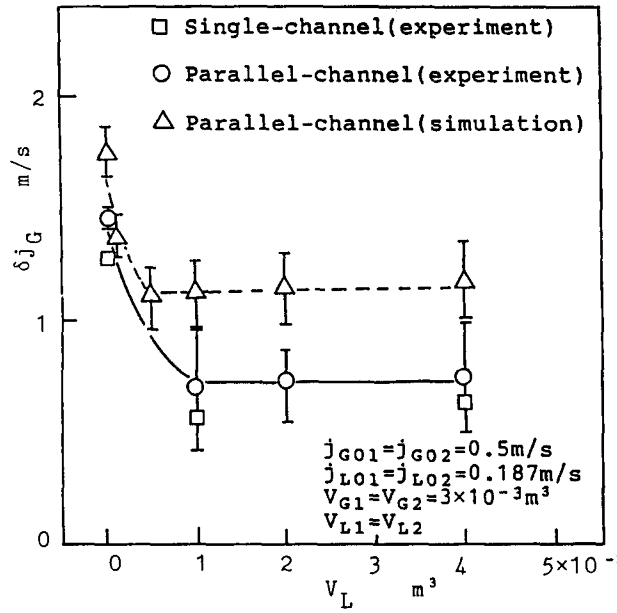


Figure 9. Amplitude of the gas flow oscillation.

The period of oscillation is also shown against the value of V_L in figure 8. The period of oscillation decreases with an increase in V_L, as discussed above, and decreases sharply at a certain value of V_L. This abrupt decrease is due to the transition of the oscillation from the relaxation type to the quasi-static type. In a quasi-static oscillation, the period becomes almost constant, independent of the magnitude of V_L.

The relationships between the amplitude of the gas flow oscillation, δj_G, and the capacity, V_L, the amplitude of the liquid flow oscillation, δj_L, and V_L, and the amplitude of the pressure drop, $\delta(\Delta P_T)$, and V_L are shown in figures 9–11, respectively. The curves in the figures represent the trends of the data points.

The value of δj_G decreases as V_L increases, almost converging in a constant value. The region of constant δj_G corresponds to the region of quasi-static oscillation. The data for a very low value of V_L correspond to the relaxation oscillation. The data for the single-channel system approximately coincide with those for the parallel-channel system.

The trends of the data point in figure 10 are contrary to those in figure 9. The value of δj_L increases from the low value of the relaxation oscillation to the higher value of the quasi-static oscillation. In figures 9 and 10, the amplitude data are widely scattered because the pressure drop characteristics differ slightly between the two channels and, therefore, the oscillation traces between two channels are not identical, as is seen in figures 5(a–d). In the transition region ($V_L = 10^{-3}$ m^3), the data is very widely scattered because of the existence of two different types of oscillation under the same operating conditions.

The value of $\delta(\Delta P_T)$ is very large in the relaxation oscillation region and very small in the quasi-static oscillation region.

On the basis of the results of the period and the amplitude of oscillation shown above we can summarize as follows: the period and amplitude of the pressure drop oscillation observed in the system with a compressible capacity in the gas feed pipe can be reduced to low levels by adding a certain compressible capacity in the liquid feed pipe, although this capacity induces a minor oscillation of the liquid flow rate.

5.3. Numerical analysis of oscillation

A numerical analysis of the flow oscillation is described in this section. An analytical model is shown in figure 12. The flow dynamics was formulated under the same assumptions as in section 4. Additionally, an isothermal change of state of the gas is assumed in the compressible capacities. Then the mass and momentum balances in the feed pipes of both phases are expressed as follows:

$$\rho_L F_t j_{L0T} \equiv \rho_L F_L j_{Li1} + \rho_L F_L j_{Li2} = \text{const}, \qquad [13]$$

$$\rho_L F_L j_{Li1} = \rho_L F_L j_{Lm1} + \rho_L F_{LC} j_{LC1}, \qquad [14]$$

$$\rho_G F_G j_{G01} = \rho_G F_G j_{Gm1} + \rho_G F_{GC} j_{GC1} = \text{const}, \qquad [15]$$

$$P_i - P_{LC1} = \frac{d}{dt}(M_{Li} j_{Li1}) + \Delta P_{Li1}, \qquad [16]$$

$$P_{LC1} - P_{L1} = \frac{d}{dt}(M_{LC} j_{LC1}) + \Delta P_{LC1}, \qquad [17]$$

$$P_{LC1} - P_{m1} = \frac{d}{dt}(M_{Lm} j_{Lm1}) + \Delta P_{Lm1}, \qquad [18]$$

$$P_{GC1} - P_{G1} = \Delta P_{GC1} \qquad [19]$$

and

$$P_{GC1} - P_{m1} = \Delta P_{Gm1}, \qquad [20]$$

where M denotes the inertial mass in each piping element, which is so small as to be neglected in the gas phase, while ΔP_L and ΔP_G represent the frictional pressure drops of the liquid and the gas in the piping elements, respectively.

The pressure–volume relations in the compressible capacities are expressed by the following equations:

$$\frac{dP_{L1}}{dt} = \left(\frac{F_{LC} P_{L1}}{V_{L1}}\right) j_{LC1} \equiv C_{L1} j_{LC1} \qquad [21]$$

and

$$\frac{dP_{G1}}{dt} = \left(\frac{F_{GC} P_{G1}}{V_{G1}}\right) j_{GC1} \equiv C_{G1} j_{GC1}. \qquad [22]$$

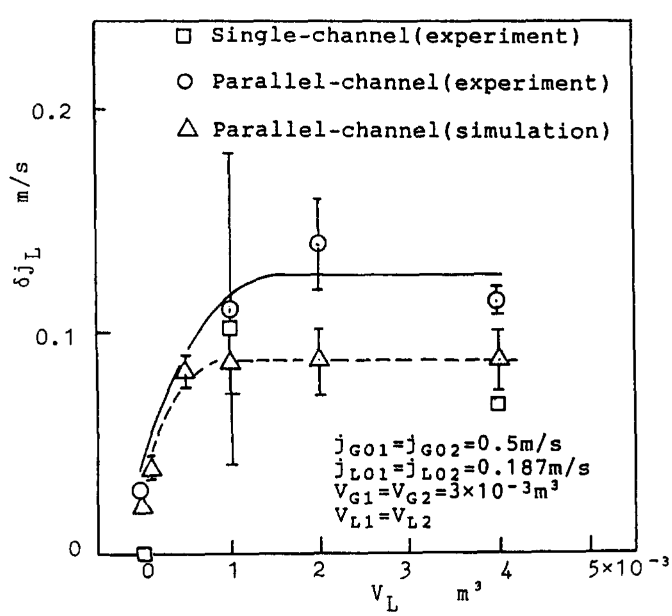


Figure 10. Amplitude of the liquid flow oscillation.

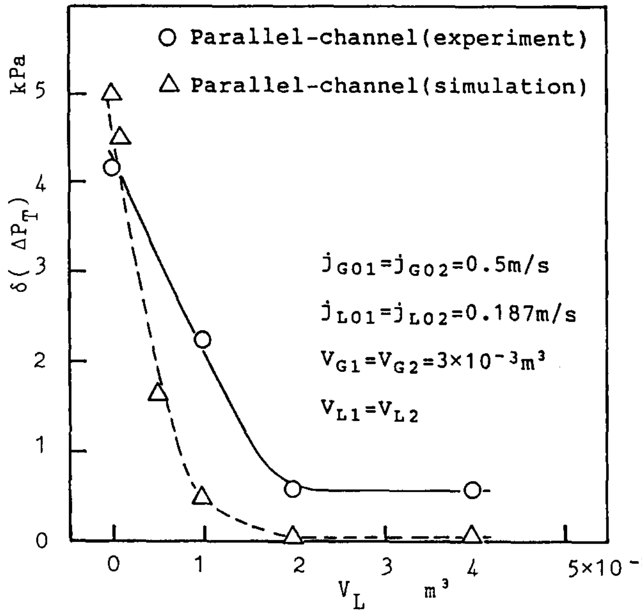


Figure 11. Amplitude of the pressure drop between the headers.

The values of C_{L1} and C_{G1} are assumed to be constant during the oscillation.

The mass and the momentum balances in the two-phase region are expressed by

$$\rho_L F_L j_{Lm1} = \rho_L F_t j_{L1}, \tag{23}$$

$$\rho_G F_G j_{Gm1} = \rho_G F_t j_{G1} \tag{24}$$

and

$$P_{m1} - P_e = \frac{d}{dt}\{M_1(j_{G1} + j_{L1})\} + \Delta P_{t1}. \tag{25}$$

The equations for channel 2 are obtained by changing the subscripts in [13]–[25] from 1 to 2.

Equations [13]–[25], and those for channel 2, were solved numerically by the Runge–Kutta method. The experimental results of the frictional pressure drops in the two-phase region were used in the calculation.

The calculated results of the oscillation of the gas flow, the liquid flow and the pressure drop for various values of V_L and a constant value of V_G are shown in figures 13(a–d). In the case of $V_L = 0$ [figure 13(a)], the oscillations of the gas flows in both channels are in phase, and the liquid flow does not oscillate. In the case of a relatively small compressible capacity in the liquid feed

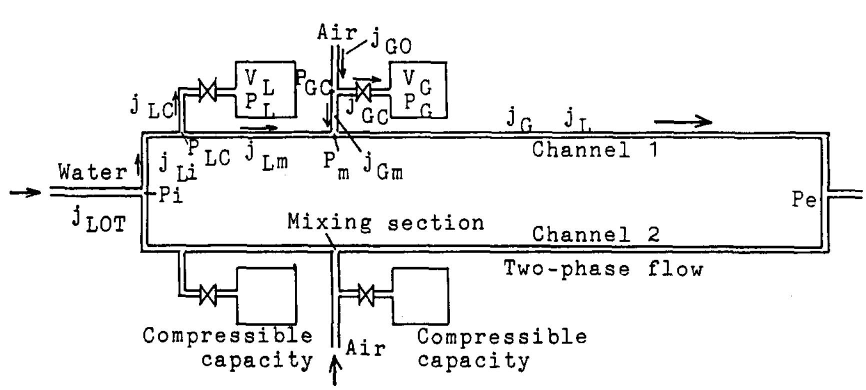


Figure 12. Analytical model.

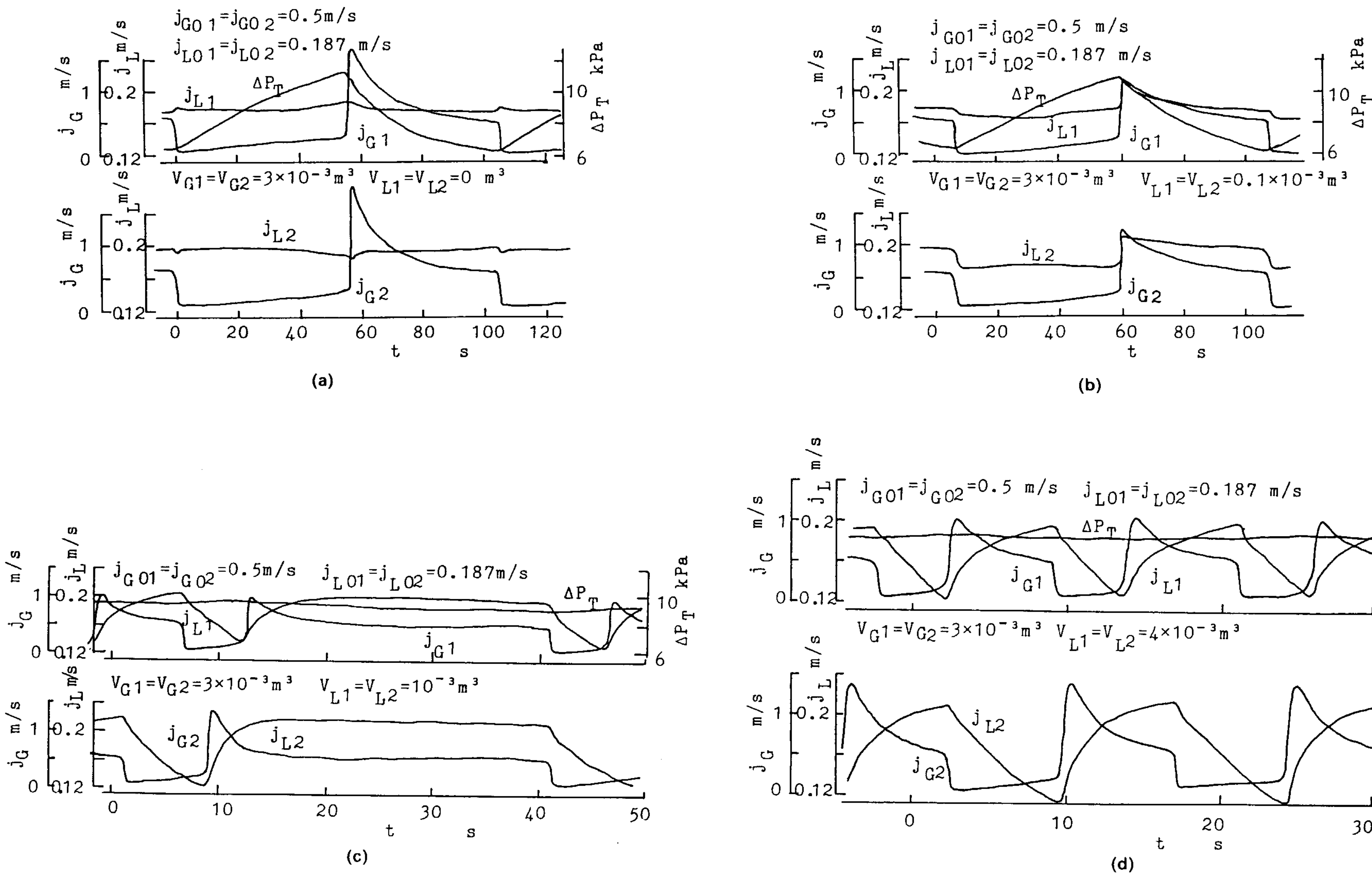


Figure 13. Oscillation traces (calculated results).

pipe, $V_L = 0.1 \times 10^{-3}$ m^3 [figure 13(b)], the oscillations of the gas flow are also in phase in both channels, and the liquid flow also oscillates in phase with the oscillation of the gas flow in each channel. The amplitude of the pressure drop between the headers is very large. In the case of $V_L = 10^{-3}$ m^3 [figure 13(c)], oscillations of j_G and j_L with a relatively long period and a relatively short period alternate. The oscillations of the gas flow and/or the liquid flow are not in phase in the two channels. The pressure drop between the headers does not oscillate. If there is a relatively large compressible capacity in the liquid feed pipe, $V_L = 4 \times 10^{-3}$ m^3 [figure 13(d)], the periods of oscillation in the channels differ and are short compared with that in case (a). There is a phase shift between the oscillations of the gas flow and those of the liquid flow. The pressure drop between the headers does not oscillate.

From these characteristics of the calculated results it can be concluded that the types of oscillation in figures 13(a, b) correspond to the relaxation oscillation while that in figure 13(d) corresponds to the quasi-static oscillations observed in the experiments. The mode of oscillation in figures 13(a, b) is the single-channel mode, that in figure 13(d) is the multi-channel mode. The oscillation obtained in figure 13(c) shows the characteristics in the transition region. The oscillation modes and types are in good agreement with the experimental results in figures 5(a–d), except for the U-tube mode.

As will be described below, the period of oscillation obtained by calculation is shorter than that observed in the experiment. Therefore, in order to compare the calculated profiles of oscillation with the oscillation in the experiments, these profiles normalized by the period of oscillation in each case are shown in figure 14. As can be seen in the figure, the calculated results are in good agreement with the experimental results for the volumetric fluxes and the pressure drop. The calculated period and amplitude of oscillation are also shown in figures 7–11. The calculated values are in good qualitative agreement with the experimental values. Therefore, the calculation produces a good representation of the characteristics of the oscillation observed in the experiment. The calculated periods of oscillation are about 40% shorter than those in the experimental results. This is mainly due to the lumped-parameter approximation of the dynamics of the system. The quantitative disagreement between experimental and calculated amplitudes also may be caused by the lumped-parameter approximation.

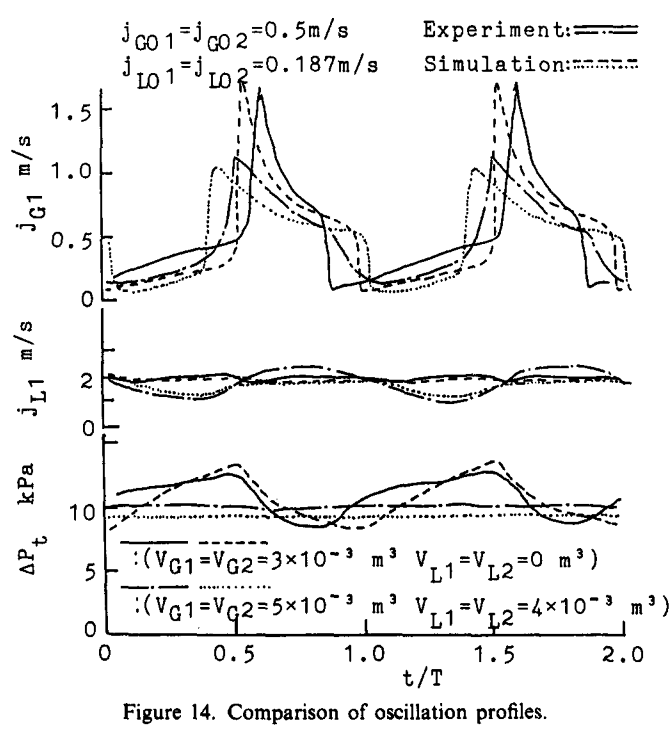


Figure 14. Comparison of oscillation profiles.

5.4. Discussion

In this section, we discuss the oscillation mode based on the analysis. As described above, the compressible capacities in the gas feed pipe act as a driving factor on the oscillation, while those in the liquid feed pipe act as a damping factor with respect to the oscillation of the gas flow. The reason is that the pressure drop has negative-slope characteristics for the gas flow and positive-slope characteristics for the liquid flow. In the equations in section 5.3, the pressure drop and the inertial terms in the single-phase region are relatively small compared with those in the two-phase region. Then the pressure differential between the headers is expressed approximately as

$$P_i - P_e \simeq P_{m1} - P_e \simeq P_{m2} - P_e.$$

Let us consider the case of a relatively large compressible capacity in the gas feed pipe, V_G, and a very small compressible capacity in the liquid feed pipe, V_L. If the pressure drop oscillation occurs in one of the channels, the pressure drop downstream of the compressible capacity in the liquid feed pipe fluctuates with a rather large amplitude, and this large pressure fluctuation is not damped by the compressible capacity in the liquid feed pipe. Consequently, this results in a large pressure fluctuation at the inlet header, which induces an in-phase flow oscillation in the other channel. This mode of oscillation is the single-channel mode. On the other hand, when there is sufficient compressible capacity in the liquid feed pipes, V_L, the compressible capacity compensates for the pressure fluctuation in the channel in a way similar to the behavior of an accumulator downstream of a plunger pump. If the pressure increases at a point downstream of the capacity, the liquid flow decreases, which results in a decrease of the pressure drop in the two-phase flow region, owing to the positive-slope characteristics of the pressure drop vs the liquid flow. Therefore, the pressure differential between the headers remains approximately constant. This situation is quite similar to the single-channel system with a large by-pass line. Then the flow oscillation is sustained independent of the other channel. This mode of oscillation is the multi-channel mode.

The U-tube mode oscillation was encountered in the experiment in the case of a medium-sized compressible capacity in the liquid feed pipe. Phenomenologically, the compressible capacity in the liquid feed pipe may be insufficient to damp the pressure fluctuation in the channel. The pressure fluctuation affects the liquid flow distribution at the Y-branch section, i.e. if the pressure drop in one channel increases, then the liquid supplied to that channel decreases. This means that the liquid flow oscillates in the U-tube mode and that this oscillation of the liquid flow induces the oscillation in the two-phase flow channel. Although the reason why the U-tube mode of oscillation is not obtained in the calculation has not yet been confirmed, it may be possible that a non-linear effect in the compressible capacity, such as the change in C_L during the oscillation, contributes to the existence of that mode of oscillation.

6. CONCLUSION

Two-phase flow systems with branch sections suffer from a non-uniform flow distribution, and from flow and pressure oscillations owing to the pressure drop charactertistics in each channel under certain operating conditions. The characteristics of the flow distribution are quite similar to those observed in boiling channels. In a system with upstream compressibility in the gas feed pipe, a flow oscillation may occur under certain conditions, depending on the pressure drop characteristics in the two-phase section. When the oscillation is the pressure drop oscillation, the amplitudes of oscillation of the gas flow and of the pressure drop can be kept small by inserting a compressible capacity in the liquid feed pipe. This compressible capacity in the liquid feed pipe has a stabilizing effect on the oscillation of the gas flow and the pressure drop, but induces a small-amplitude oscillation of the liquid flow. The types of pressure drop oscillation encountered in parallel-channel systems are relaxation oscillation and quasi-static oscillation. In the parallel-channel system, various modes of oscillation occur, i.e. a single-channel mode, a U-tube mode and a multi-channel mode, depending on the magnitudes of the compressible capacities in the gas and the liquid feed pipes.

The non-uniform flow distribution and the oscillations in the parallel-channel systems are analyzed by means of a lumped-parameter model of two-phase flow dynamics. The results of these analyses qualitatively are in good agreement with the experimental results. Thus, the characteristics of flow distribution and oscillation can be estimated by the analytical approach presented in this paper.

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